**Asset Pricing Models**

**Capital Asset Pricing Models (CAPM)**

# Key Assumptions

* **Individual Behaviour:**
  + Investors are **mean-variance optimizers** with the **same investment horizon**
  + Investors have **homogenous expectations** – same estimates **of inputs**
    - In practice, this assumption is *too strict****,*** thus we instead assume that investors have **rational expectations** - correctly interpreting information
* **Market Structure**
  + All assets are **available** to trade with **no taxes/transaction costs**
  + Investors can **lend/borrow** at the risk-free rate as well as **short sell** assets

# Capital Market Line (CML)

* Given the key assumptions described earlier, this means that **ALL** investors have the **same Efficient Frontier** and **Capital Allocation Lines**
* Additionally, since ALL assets are **available** for trade, the optimal risky portfolio will always be the **Market Portfolio –** containing **ALL risky assets** in the market
  + Adding additional assets to the portfolio has marginal reductions in risk but the risk is still reduced. Given that the market portfolio has the **lowest risk** & the **same expected return**, it will always have the **highest Sharpe Ratio - Optimal**
* To distinguish this special case, we call the CAL that uses the Market Portfolio the **Capital Market Line** (CML) instead
* Thus, combining these two premises, we reach the conclusion that every investor invests in **some combination of a Risk-Free Asset & Market Portfolio**

# Market Portfolio

* Contains **ALL possible securities** weighted by their **relative Market Values**
  + If an asset is not included, it means that **investors will short** it, causing its price to fall eventually **until it becomes attractive** to be added to the portfolio
* When prices change, the portfolio will **automatically rebalance** itself. Since it requires **little trades to maintain**, it is known as a **Passive Portfolio**
* Given that the market portfolio is the efficient portfolio (Nothing better than it), any attempts to outperform it (**Active Portfolios**) are **futile** as the **returns are lower** and **significant research cost** was spent on it. **Passive > Active Portfolio**

## Why is the optimal portfolio the Market portfolio?

* All investors have the **same optimal risky portfolio** with a **risk-free asset**
  + Some people borrow while some people lend – in aggregate, these people will **offset** each other as a lender needs a borrower, making **total investment in the risk-free asset 0**
* This leaves only the **Optimal Risky Portfolio demanded by investors in aggregate**
* The aggregate supply of investments is simply **all possible securities – Market Portfolio**
* To be **in equilibrium**, Demand must equal to Supply – thus the Optimal Risky Portfolio demanded **must be the Market Portfolio**

This has three key implications:

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* The combination of all investors must be **equivalent to the Market**

# Reward to Risk Ratio

* Ratio of how much an asset contributes to Risk Premium (Reward) to how much it contributes to the Variance (Risk) of the **Market Portfolio**
* In equilibrium, all assets within the Market Portfolio must have the **same RRR**:
  + **Higher RRR** → Investors will buy more of it, driving up the price till it has the same RRR as the market portfolio
  + **Lower RRR →** Investors will sell more of it, driving down the price till it has the same RRR as the market portfolio
* By equating the RRR of an individual asset and that of the market, we can rearrange to get an expression for the **expected return of any asset – Capital Asset Pricing Model**

## Reward Contribution

* We **focus on each assets Risk Premium** rather than absolute return
* An assets contribution to the Reward is thus the product of the **weight of the asset and its Risk Premium**



## Risk Contribution

* Recall from the Markowitz model that the Variance of the portfolio is the **sum product of the weights and covariances**
* An assets contribution to the Risk is thus the sum product of the **weights and Covariance terms that contain the asset**





# Capital Asset Pricing Model

* Re-arranging the Reward to Risk Ratio, we obtain the CAPM model:

















## Interpreting Beta

* Beta is a measure of the **systematic risk of an asset -** sensitivity of the asset's return to the return of the market
* Formally defined as the change in asset return **per 1% change** in market return
* Beta can be determined by **regression**, where Beta is the slope of the regression:
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* Since the market is made up of stocks, the average Beta of a stock should be 1
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* Since unsystematic risk is random, they cancel out in the long run thus have **0 expectation**
* Similarly, since they are random, they are **uncorrelated with systematic risk** thus have **0 covariance**
* For a sufficiently well-diversified portfolio, the unsystematic risk **tends to 0** in the long run, **leaving only the systematic market risk**

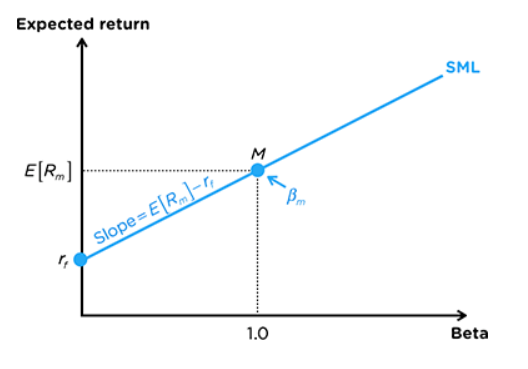




## Security Market Line (SML)

* We can represent the CAPM model graphically through the **Security Market Line**
* The key takeaway is that the **Expected Return** on an asset **depends ONLY on the Systematic Risk** (Measured through Beta)
  + CAPM can also be used to measure a **non-market portfolio’s expected return**
  + Use the Beta of the portfolio instead, which is weighted average of individual Betas
* Alpha is the **difference between the expected return of an asset and the required return by CAPM**
  + Graphically, assets with positive/negative alpha lie **above/below the SML**
  + Alpha is the **vertical distance** between the point and the SML

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| --- | --- |
| **Capital Market Line** | **Security Market Line** |
| Result of Markowitz Optimization | Result of CAPM |
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### Key Understanding

* **CAPM** measures the required return based on its systematic risk - **Financial Approach**
* **Expected Return** measures the return based on a historical returns - **Statistical Approach**
* Both approaches are similar:
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## Equilibrium under CAPM

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* In other words, the Market Portfolio is **no longer efficient** because there **exists a better portfolio** that added more/less of that asset
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* This requires all investors to do the same – **each investor making small changes** in their portfolio to restore CAPM equilibrium
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## Adding a new Investment

* We can decide if we want to add a **new investment** into the market portfolio by using CAPM
* If the new investment provides an expected return higher than that of what is predicted by CAPM, then it should be added into the portfolio
* The converse is true for selling; if the Sharpe Ratio is lower, then the asset should be sold









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**Arbitrage Pricing Model**

# What is Arbitrage?

* Arbitrage is a situation where you can make **riskless profits** without a net investment
  + Typically occurs when assets are mispriced across different markets
  + Exploit arbitrage by **buying low (Cheaper) and selling high (Expensive)**
* Naturally, we **assume that no arbitrage exists** in the market – thus assets with the same payoff will have the same price (Law of one price)
* In the event there is arbitrage, the very act of **exploiting arbitrage helps to remove it**:
  + Buying cheap (under-priced) assets raises its price (Lowers return)
  + Selling expensive (over-priced) assets lowers its price (Raises return)
  + This occurs till all assets are fairly priced, eliminating the arbitrage opportunity
* This sounds similar to the CAPM model with one key difference – Since arbitrage is riskless, arbitrageurs will want to **take a large position** in it to maximize profits, thus **very few investors are needed to restore arbitrage equilibrium**

# Arbitrage Pricing Model

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  + If they have different returns, you could long the asset with higher returns and short the one with lower returns to create exploit arbitrage
* To be even more precise, **all assets must have a return proportional to their beta**
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  + In that case, both the portfolio and the other asset must have the **same return**
  + The portfolio’s return lies on a **straight line from the risk-free rate**. Thus, for no arbitrage to occur, all asset’s return must **lie on this line as well**
* In effect, this creates a situation identical to the **Security Market Line & hence CAPM** model, but **through different assumptions**

Thus, for a well-diversified portfolio that tracks the market,



# Well-Diversified VS Efficient

* A **Well Diversified portfolio** contains **ONLY systematic risk**
* An **Efficient Portfolio** has the **highest Sharpe Ratio** - All else equal, it means that it should have the lowest possible risk as well (Systematic Risk ONLY)
  + An Efficient Portfolio is always well-diversified
  + But not all well-diversified portfolios are efficient - only the portfolio with the **highest return per unit risk**

# Comparison to CAPM

* CAPM makes a strict assumption that the Market Portfolio is efficient and that an efficient portfolio is required to calculate the return of a stock
* In practice, it is often **hard to find** a single efficient portfolio that captures ALL the systematic risk because it is hard to properly measure risk and return
* The APT thus uses **one or more well-diversified portfolios** that each capture some part of the systematic risk and **together forms an efficient portfolio** which captures the entire systematic risk
  + Note that well-diversified portfolios typically only capture some part of the systematic risk
  + Since they do not consider all factors, they *may* leave some unsystematic risk behind

**Multi Factor Models**

# Multi Factor Models

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* The issue is that it **oversimplified systemic risk** – it failed to capture risks that affected certain industries only
* A more realistic approach would be to **further break Systemic Risk** down into sub-components – Inflation, Interest Rate, GDP etc
* Each of these factors will **contribute a different amount** to the overall return; each with their own respective **Factor Beta**
  + In order to measure the Returns of a Factor and the Beta with respect to those returns, we can use a **well-diversified portfolio that tracks that specific factor**
  + Note that these are based off the **Arbitrage Pricing Model**





## Self-Financing Portfolios

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* The portfolio is **funded through borrowing** at the Risk Free Rate
  + Short Treasury Bills at the Risk Free Rate
  + Use the short sale proceeds to Long the Well-Diversified Factor Portfolio
  + Pay back the Risk Free Rate - Earn the **Risk Premium**
* Thus, the Factor portfolio is **financed externally - NOT SELF FINANCED**
* Given a generic multi-factor portfolio, we should always assume that they are **NOT self-financed**

## Interpretation of Beta

* We can think of CAPM as a special case of CAPM with only one factor - **Market Risk**
* Thus, we can generalize Beta to refer to the **Factor Portfolio rather than the Market Portfolio**







## Comparison to Single Factor Portfolios

* They are **better than Single Factor** because it is **easier to find** multiple well-diversified portfolios that together capture systematic risk rather than just a single efficient portfolio that captures it
* They are **worse than Single Factor** because it is more **tedious to measure** the return of multiple well diversified portfolio rather than just a single efficient portfolio
* There is **no clear answer** as to which is better

# Farma French Carhart Four Factor Model

|  |  |  |
| --- | --- | --- |
| **Market Factor** | Firms perform in **proportion to the market**   * Measured through Beta |  |
| **Size Factor** | **Small Firms** tend to perform better than Large ones | **Small Minus Big** (SMB) |
| **Value Factor** | **Value Firms** tend to perform better than Growth ones   * By Book (Accounting) to Market Ratio | **High Minus Low** (HML) |
| **Momentum Factor** | **Winning Firms** tend to perform better than Losing ones   * By past year performance (Top 30% of firms) | **Up Minus Down** (UMD) |



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## Self-Financing Portfolios

* Notice that these Factors do not borrow at the risk free rate
* The factor portfolios themselves go Long and Short something else - **Self Financing**
* The return is thus the **excess return of the winners over the losers - Factor Premium**
* **ONLY** the FFCH is self-financing (No need to subtract the risk-free rate)
* Each Self Financing portfolio has a **weight that sums to 0** - Long and Short positions offset each other



## Book to Market Explanation

* **Book Value** represents the **objective value** of the firm measured through Accounting
* **Market Value** represents the **subjective/Market value** of the firm measured through Share Price
* The Book to Market Ratio measures the relative size of the two:
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  + 
* **Value Firms** are firms with a **ratio greater than 1**; they are currently undervalued and thus should be purchased to allow for returns once the market correctly values them
* **Growth Firms** are firms with a **ratio smaller than 1**; they are currently overvalued as the investors **anticipate** large growth in the future